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EXPERIMENTS ON THE BREAKDOWN OF LAMINAR FLOW IN A PARALLEL-PLATE CHANNEL

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Abstract—The effect of entrance section geometry and superposed sources of disturbance on the minimum Reynolds number marking the breakdown of laminar flow was investigated experimentally. Twelve different flow configurations were employed throughout the course of the study. It was found that the presence of stationary sources of disturbance in the upstream plenum chamber and in the entrance section does not have a significant effect on the breakdown Reynolds number. A pulsating disturbance has a greater influence on the breakdown of the laminar regime, as does asymmetric entrance section geometry. The lowest value of the breakdown Reynolds number encountered in these studies was 2200, corresponding to a disturbance source situated within the channel itself.

NOMENCLATURE

A,	aspect ratio of channel cross-section;
D_e,	equivalent diameter;
f,	friction factor;
p,	pressure;
Re,	Reynolds number;
V,	mean fluid velocity;
x,	axial coordinate along channel;
$(f \cdot Re)^*$,	theoretical value of friction factor–Reynolds number product for laminar flow;
ρ,	density;
ν,	kinematic viscosity.

INTRODUCTION

THIS PAPER is concerned with the experimental exploration of the minimum Reynolds number for the breakdown of laminar flow in a parallel-plate channel. To this end, a test apparatus incorporating interchangeable entrance section geometries and sources of disturbance was employed, with water as the working fluid. All together, twelve different entrance and/or disturbance configurations were utilized during the course of the experiments.

Prior investigations relevant to the breakdown of laminar flow in a parallel-plate channel have been reported by Davies and White [1] and by Whan and Rothfus [2]. In each case, the experiments were limited to a single duct configuration, with no attention given to different entrance section geometries and disturbance sources. The results of Davies and White were based on the readings of a single pair of pressure taps, with the upstream tap situated in the hydrodynamic development region for most of the test conditions. As a consequence, their findings must be viewed with some uncertainty.* The inlet configuration of the Davies–White apparatus was such that the flow entered the channel at right angles to the longitudinal axis of the channel. The experiments of Whan and Rothfus were conducted in an apparatus with a smooth, tapered inlet section.

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* Another somewhat more subtle error in the work of Davies and White stems from the fact that their duct heights were inferred from a comparison of measured friction factors with that for laminar flow in a parallel-plate channel. Even for a rectangular duct with an aspect ratio of 50, the friction factor, Reynolds number product is 2·7 per cent below that for the parallel-plate channel.

In the presentation that follows, the experimental apparatus will be described first, with particular attention to the various entrance section geometries and sources of disturbance. The experimental findings are presented and discussed in the last section of the paper.

EXPERIMENTAL APPARATUS

Flow system components and instrumentation. The experimental facility, shown in Fig. 1, is a closed-loop system which includes, along the path of flow, the upstream plenum chamber, test section, downstream plenum chamber, metering station, sump, pump and filter. The system was designed to provide finely controlled, steady operating conditions and to facilitate measurements of high accuracy. The working fluid is distilled water.

The upstream plenum serves the dual functions of a flow control device and a calming chamber. The liquid level in the plenum can be set and maintained at any desired height by means of an overflow weir which is adjustable to any vertical position. By this means, the upstream plenum functions as a constant head tank. The calming is accomplished by the substantial volume of the plenum and by three stainless steel screens.

The test section is a horizontal channel of rectangular cross section. The height of the channel can be varied by the use of precisely machined spacer strips. For the Reynolds numbers reported herein, channel heights ranged from 0·051 to 0·1193 in. The channel width is 3·5 in., resulting in aspect ratios (width to height) ranging from 69 to 29. Thus, practically speaking, the test section may be regarded as a parallel-plate channel. The streamwise length of the channel is approximately 16½ in., so that, for the present test conditions, the ratio of channel length to height ranged from 325 to 140. These length–height ratios are sufficiently great to insure that hydrodynamically developed flow prevails in a substantial portion of the channel.

The lower wall of the channel is of aluminum, the surface of which had been hand polished to a high degree of smoothness. The upper wall is

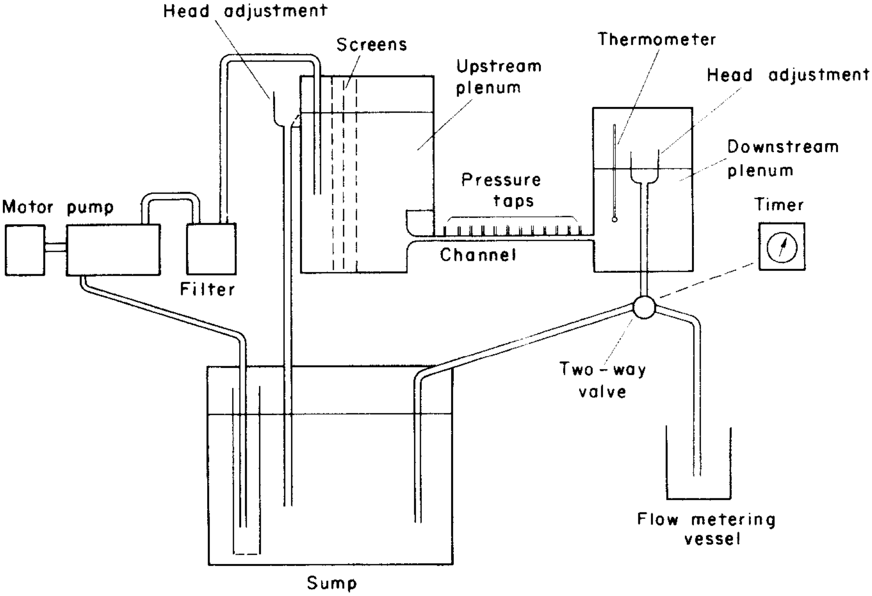

FIG. 1. Schematic of the experimental apparatus.

an optically flat, ground glass plate. Pressure taps were painstakingly installed along the length of the upper wall. The taps are formed from 0·031 in. i.d. brass tubes, lapped flush with the glass. Of the 29 pressure taps, 7–10 typically fell within the region of hydrodynamically developed flow. The distribution of static pressure along the test section was displayed on a glass tube manometer bank. The liquid levels in the manometer tubes were read with a Gaertner cathetometer capable of discriminating to within 0·05 mm (0·002 in.).

The downstream plenum is also fitted with an overflow weir that can be positioned so as to fix the liquid level in the plenum at any desired height. Independent adjustment of the overflow weirs in the upstream and downstream plenums thus permits the specification of the pressure difference between the inlet and outlet cross sections of the test section.

Upon leaving the downstream plenum, the working fluid passes through a two-way valve, one exit of which leads to the flow metering station while the other exit leads to the sump. Metering was accomplished by direct weighing of quantities of working fluid, typically on the order of 4000 g. For this purpose, a precision beam balance capable of being read to 0·1 g was employed. The time required to accumulate the aforementioned samples was measured by a timer driven by a synchronous electric motor, the timer being automatically actuated by the flow control valve and capable of being read to 0·05 s.

Except for the short periods devoted to metering, the flow was normally ducted to the

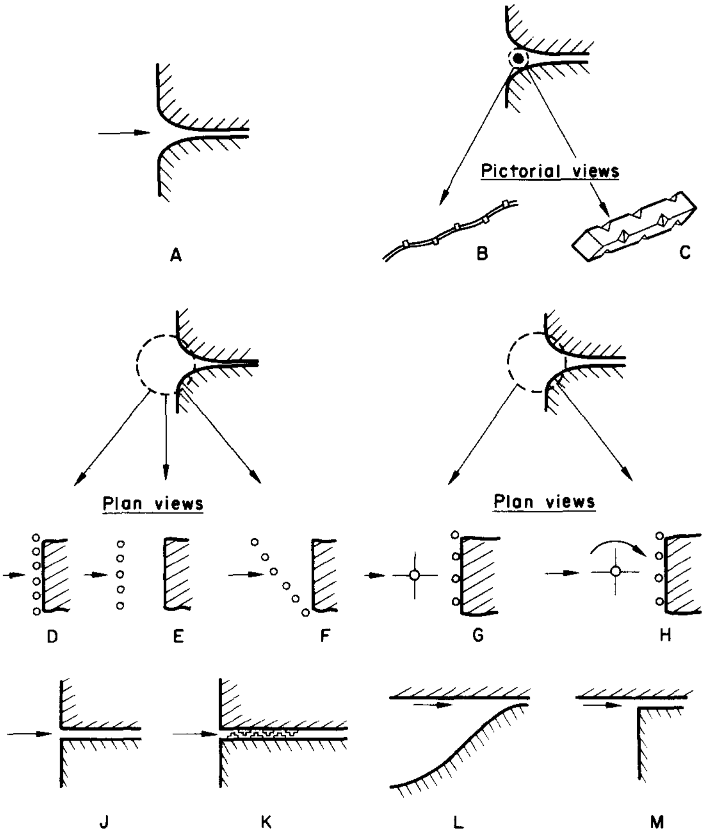

FIG. 2. Schematic representations of the flow configurations.

sump. The pump, Moyno model 1L4, employs a novel rotating screw action to provide a steady continuous discharge, free of obvious pulsations. A 5 μ filter is situated in the line between the pump outlet and the upstream plenum.

Entrance section geometries and disturbance sources. The description of the various inlet geometries and disturbance sources is facilitated by reference to Fig. 2. The configurations for which experiments were performed are indexed by the letters A, B, . . . , M. Configurations A through H utilize a symmetric, rounded entrance section formed from quarter ellipses with semi-major and semi-minor axes of one in. and $\frac{1}{2}$ in. The rounded entrance section itself, without superposed sources of disturbance, is characterized as configuration A. All of the configurations B–H include sources of disturbance. In B, a slightly undulating wire (dia. = 0·050 in.) with periodic protrusions* is positioned in the contraction section, spanning the width of the channel. Configuration C utilizes a similarly located disturbance source which consists of a $\frac{1}{8}$ in. square rod with periodic, $\frac{1}{16}$ in. deep notches. In each of the foregoing cases, the disturbance source filled about half of the cross section in which it was situated.

Configurations D, E and F incorporate a grid of vertical rods situated in the upstream plenum. Each rod is $\frac{1}{16}$ in. dia. and 3 in. long, with a center-to-center distance between rods of $\frac{1}{8}$ in. In D, the grid is positioned immediately adjacent to the inlet of the contraction section, while in E, the grid is situated about one in. upstream of the plane of the inlet. For configuration F, the vertical plane containing the ensemble of rods is oriented at an angle of 30° with the plane of the inlet section.

Configurations G and H include a flat-bladed stirrer, in addition to the just-discussed grid of vertical rods, now modified so that the center-to-center distance is $\frac{3}{8}$ in. The stirrer consists of a vertical shaft (dia. = 0·3 in.) to which are affixed four blades, each $\frac{7}{8}$ in. in length and 0·2 in. in height. The axis of the stirrer is 2 in. upstream of the plane of the inlet section, while the grid of vertical rods is immediately adjacent to the inlet. Configuration G corresponds to the stirrer in place, but not rotating, while configuration H corresponds to the stirrer rotating at 300 rpm.

Both configurations J and K utilize a symmetric, square-edged entrance section. The first of these is the square entrance itself, without superposed disturbances, while the second includes a disturbance source located *within* the channel. The disturbance source was fabricated from $\frac{1}{8}$ in. wide strips of plastic tape which spanned the entire $3\frac{1}{2}$ in. channel width. Three such strips, spaced $\frac{1}{8}$ in. apart, were attached to each wall in a staggered arrangement as illustrated in the sketch. The thickness of the plastic tape was 0·01 in. Periodic protuberances, 0·017 in. in height (relative to the tape surface), 0·05 in. in width, and 0·1 in. in length, were punched into the tape by means of a letter punch using the hyphen symbol. There were six such protuberances per inch of tape length.

The last two configurations, L and M, are characterized by asymmetric entrance sections. In the former, the curved lower wall reduces the cross sectional height by about $1\frac{1}{2}$ in. in an axial length of $2\frac{1}{2}$ in. The latter utilizes a square-edged contraction at the lower bounding wall.

RESULTS AND DISCUSSION

The measured distributions of static pressure along the length of the channel were utilized to determine axial pressure gradients $\mathrm{d}p/\mathrm{d}x$ for the hydrodynamically developed regime (characterized by a linear relationship between p and x).† Numerical values of $\mathrm{d}p/\mathrm{d}x$ were evaluated by fitting a least-squares straight line through the relevant data. The mean velocities V were calculated from the measured mass flow rates.

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[*] Formed by globules of solder.

[†] The region of hydrodynamically developed flow occupied most of the channel length. For instance, the entrance length for configuration A is approximately given by the relation $x/D_e = 0{\cdot}015\,Re$.

With these quantities, the friction factor f and Reynolds number Re follow directly from their definitions

$$f = \frac{(-\mathrm{d}p/\mathrm{d}x)\, D_e}{\frac{1}{2}\rho V^2}, \qquad Re = \frac{V D_e}{\nu} \tag{1}$$

in which D_e is the equivalent diameter, and ρ and ν respectively represent the fluid density and kinematic viscosity.

For large aspect ratio rectangular ducts, analysis shows that the product of the friction factor and the Reynolds number can be expressed as

$$(f \cdot Re)^* = \frac{96}{1 - (192/A\pi^5)} \left(\frac{A}{A+1}\right)^2 \tag{2}$$

where A is the aspect ratio of the rectangular cross section. The asterisk is appended to facilitate distinguishing the analytical results given by equation (2) from those of experiment.

The behavior of the $f \cdot Re$ product with Reynolds number is a sensitive indicator of the

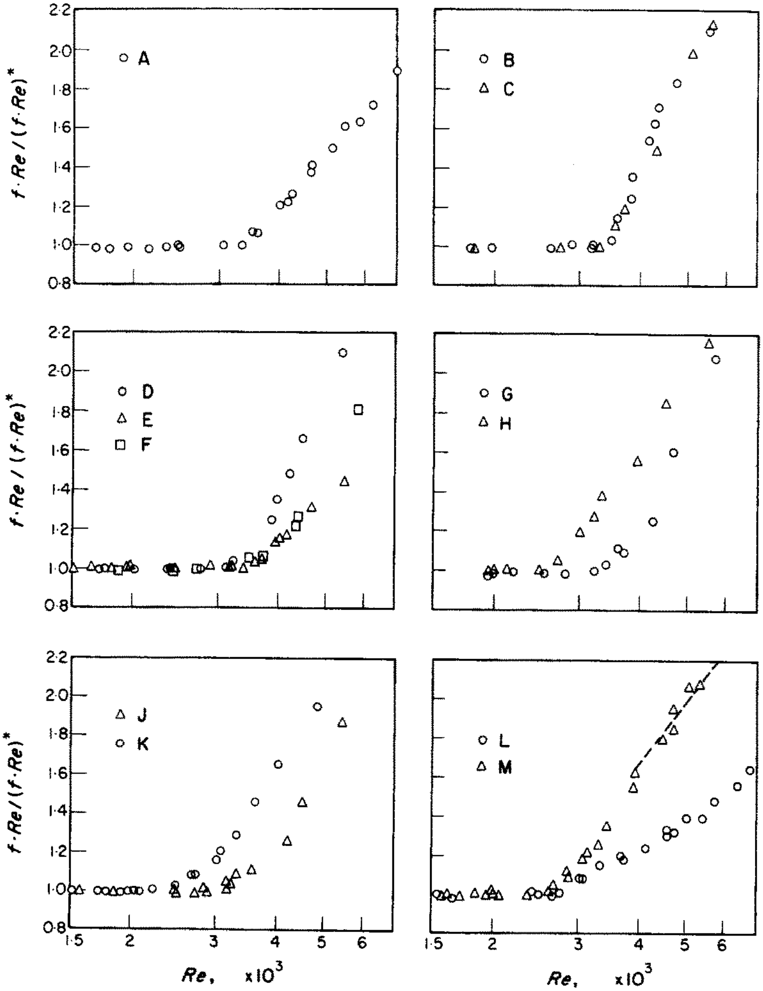

FIG. 3. Reynolds number dependence of the friction law for parallel-plate channels with various entrance section geometries and disturbance sources.

breakdown of the laminar regime. To this end, the ratio $(f \cdot Re)/(f \cdot Re)^*$ is plotted as a function of the Reynolds number. In this ratio, the numerator is from experiment while the denominator corresponds to the prediction of laminar theory. For laminar flow conditions, the $(f \cdot Re)/(f \cdot Re)^*$ ratio should be unity, within the limits of experimental precision. With the breakdown of the laminar regime, this ratio will take on values greater than unity.

The results for the $(f \cdot Re)/(f \cdot Re)^*$ ratio are plotted in Fig. 3, with the Reynolds number as abscissa variable. The figure is subdivided into six separate grids, which correspond to the six groupings shown in Fig. 2. The Reynolds number range extends from about 1500 to 7000.*

Inspection of the figure shows that the laminar and non-laminar regimes are clearly distinguishable. For the former, $(f \cdot Re)/(f \cdot Re)^*$ is essentially unity and independent of the Reynolds number. For the latter, $(f \cdot Re)/(f \cdot Re)^*$ exceeds unity and increases with the Reynolds number. In the laminar range, the majority of the data points are within one per cent of the value $(f \cdot Re)/(f \cdot Re)^* = 1$, thereby testifying to the high precision of the experiments.

Table 1. Reynolds numbers marking the breakdown of laminar flow

Configuration	*Re*	Configuration	*Re*
A	3400	G	3300
B	3400	H	2600
C	3400	J	3100
D	3200	K	2200
E	3400	L	2700
F	3400	M	2600

The Reynolds numbers corresponding to the breakdown of the laminar flow regime can be identified by examination of Fig. 3. This information is listed in Table 1 for each of the experimental configurations. It is seen from the table that the breakdown Reynolds number ranges from 3400, which corresponds to several configurations involving the rounded inlet section, to 2200 for configuration K, for which the disturbance source is situated within the channel.

Several interesting characteristics emerge from a careful scrutiny of Table 1. First of all, the presence of stationary disturbance sources in the plenum chamber or in the contraction section (configurations B, C, D, E, F and G) do not have a significant effect on the breakdown Reynolds number. It may be hypothesized that disturbances from such sources are damped as the flow passes through the contraction section. A pulsating disturbance (configuration H) is somewhat more effective in causing breakdown of the laminar flow. Entrance section geometry (configurations A, J, L and M) also has a moderate influence in reducing the breakdown Reynolds number, with asymmetry (configurations L and M) playing a greater role than squaring of the contraction section (configuration J).

Configuration K is unique among those investigated in that the disturbance source is situated within the channel. Disturbances generated by such a source cannot, therefore, be damped by the contraction of the flow as it passes through the channel inlet section. The breakdown Reynolds number for configuration J, namely 2200, is the lowest of those encountered in these tests, and this is consistent with the foregoing discussion. It should be noted, however, that the value of 2200 for the breakdown Reynolds number may not necessarily be the absolute lower limit for the breakdown of laminar flow in a parallel-plate channel. On the other hand, it is the lowest value thus far reported in the literature.

It is interesting to inquire as to how the present non-laminar results compare with standard correlations for turbulent flow. If one employs the Blasius formula, then

$$\frac{f \cdot Re}{(f \cdot Re)^*} = \frac{0{\cdot}316\, Re^{0{\cdot}75}}{(f \cdot Re)^*}. \tag{3}$$

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* Data were actually collected over a much larger range, but are not included herein because of space limitation.

For purposes of illustration, equation (3) has been plotted as a dashed line in the lower right-hand grid of Fig. 3 for $Re \geqslant 4000$. It is seen that good agreement between the correlating equation and the present data exists for those configurations which favor the early development of fully turbulent conditions.

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REFERENCES

1. S. J. DAVIES and C. M. WHITE, An experimental study of the flow of water in pipes of rectangular section, *Proc. R. Soc.* **A119**, 92–107 (1928).
2. G. A. WHAN and R. R. ROTHFUS, Characteristics of transition flow between parallel plates, *A.I.Ch.E. Jl* **5**, 204–208 (1959).

EXPÉRIENCES SUR L'ARRÊT DE L'ÉCOULEMENT LAMINAIRE DANS UNE CONDUITE À PLAQUES PARALLÈLES

Résumé—L'effet de la géométrie de la section d'entrée et des sources de perturbation superposées sur le nombre de Reynolds minimal marquant l'arrêt de l'écoulement laminaire a été étudié expérimentalement. Douze configurations différentes d'écoulement ont été employées tout au long de cette étude. On a trouvé que la présence de sources stationnaires de perturbations dans la chambre de tranquillisation amont et dans la section d'entrée n'a pas d'effet important sur le nombre de Reynolds de transition. Une perturbation pulsatoire a une influence plus grande sur l'arrêt du régime laminaire, ainsi qu'une géométrie asymétrique de la section d'entrée. La valeur la plus faible du nombre de Reynolds de transition rencontrée dans ces études était de 2200, correspondant à une source de perturbation située dans la conduite elle-même.

VERSUCHE ÜBER DEN UMSCHLAG LAMINARER STRÖMUNG IN EINEM EBENEN PARALLELKANAL

Zusammenfassung—Es wurde der Einfluss von Einlaufgeometrie und aufgeprägten Störquellen auf die minimale Reynoldszahl des laminaren Umschlags untersucht. Während der Untersuchungen wurden zwölf verschiedene Strömungsformen verwendet. Man fand, dass das Vorhandensein stationärer Störquellen in der Beruhigungskammer stromaufwärts und im Einlauf keinen wesentlichen Einfluss auf die Reynoldszahlen des Umschlags hat. Eine pulsierende Störung hat einen grösseren Einfluss auf den Umschlag als eine asymmetrische Einlaufgeometrie. Der kleinste Werte der Reynoldszahl des Umschlags war bei diesen Untersuchungen 2200, entsprechend einer Störquelle im Kanal selbst.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ ПОТЕРИ УСТОЙЧИВОСТИ ЛАМИНАРНОГО ТЕЧЕНИЯ В ПЛОСКО-ПАРАЛЛЕЛЬНОМ КАНАЛЕ

Аннотация—Экспериментально исследуется влияние геометрии входного участка и введенных в поток источников возмущения на минимальное значение критерия Рейнольдса, характеризующего разрушение ламинарного течения. Исследовались 12 различных конфигураций потока. Найдено, что наличие стационарных источников возмущения в успокоительной камере и на начальном участке не оказывают значительного влияния на критическое значение критерия Рейнольдса. Пульсирующее возмущение оказывает сильное влияние на разрушение ламинарного режима так же, как и асимметричная геометрия входного сечения. Минимальное значение найденного в опытах критического значения критерия Рейнольдса составляло 2 200.